Theorem: for any n $n^2 + n + 1$ will always be odd

Proof: Take an arbitrary n. n will either be odd or even. If n is even that means n = 2k. So n^2+n+1 will be $4k^2+2k+1$ after substitution. $2(2k^2+k)+1$ is clearly an odd number so for all even n the theorem holds. If n is odd that means n=2k+1 so n^2+n+1 will be $(4k^2+4k+1)+(2k+1)+1 = 2(2k^2+3k+1)+1$ after substitution. This is clearly an odd number so the theorem holds for all odd n. This implies that for all n: $n^2 + n + 1$ will always be odd and so the theorem is proven.